MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2017/2018

DIM5068 – MATHEMATICAL TECHNIQUES 2

(T71)

25 OCTOBER 2017

02.30 pm – 04.30 pm

(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **2** pages with **4** questions.
2. Attempt **ALL FOUR** questions. The marks distribution for each question is given.
3. Write your answers in the answer booklet provided.
4. Key formulae are given in the Appendix.

Question 1

a) Given $Z_1 = a + 4i$ and $Z_2 = 1 - bi$ are two complex numbers such that, $Z_1 + Z_2 = 1$. Find the values of a and b. (4 marks)

b) Find the complex zeros of the polynomial functions, $f(x) = x^3 - 5x^2 + 2x - 10$ and rewrite the function in factored form. (6 marks)

[TOTAL 10 MARKS]

Question 2

a) Differentiate $y = 7x^7 + 7\sec x + 7\ln x - 7$. (2 marks)

b) Given the function of $y = \dfrac{3}{2(9x+3)^2}$.

i. Find the first derivative by using chain rule. (4 marks)

ii. Show that $\dfrac{dy}{dx} = -1$ when $x = 0$. (2 marks)

c) For the function $f(x) = 2x^3 + 4x^2 + 2x$,

i. find the critical number(s). (4 marks)

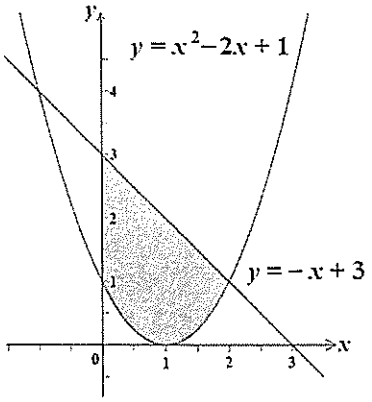
ii. identify the intervals on which f is increasing or decreasing. (3 marks)

iii. determine the maximum and/or the minimum value(s). (2 marks)

d) Integrate $\int (2x + 8e^x - 5)dx$. (2 marks)

e) By using substitution rule, integrate $\int \dfrac{6x^3}{(x^4 - 7)^{10}} dx$. (5 marks)

f) Find the area of a farm as shown by the shaded region below, which is bounded by $y = x^2 - 2x + 1$, $y = -x + 3$ and the sides from $x = 0$ until $x = 2$. (6 marks)



[TOTAL 30 MARKS]

Continued...

Question 3

a) Solve for y, given that the first order differential equation is $\frac{dy}{dx} = \frac{12x^2 + 4x}{y}$, by using the **Separable Method**. (5.5 marks)

b) Given the differential equation $\frac{dy}{dx} + \frac{y}{x} = \cos x$.

i. Identify the $p(x)$ and $q(x)$. (2 marks)

ii. Calculate the integrating factor, μ. (2 marks)

iii. Find μy given $\mu y = \int \mu q(x) dx$. [Hint: Use integration by parts] (5.5 marks)

iv. From your answer in (iii), determine the solution of the initial value problem if $y(0) = 1$. (2 marks)

v. State the general solution of y. (1 mark)

c) Given the non-homogeneous differential equation $y'' - 8y' + 15y = 75x - 25$.

i. Determine the complementary solution, y_c. (4 marks)

ii. Compute the particular solution, y_p. (8 marks)

[TOTAL 30 MARKS]

Question 4

a) Given that $\boldsymbol{a} = 2i - j - 2k$ and $\boldsymbol{b} = 12i - 5j$. Find,

i. $3\boldsymbol{a} + 2\boldsymbol{b}$. (4 marks)

ii. a unit vector that has the same direction as $\boldsymbol{a}$. (3 marks)

iii. a vector orthogonal to both $\boldsymbol{a}$ and $\boldsymbol{b}$. (4 marks)

b) Find a vector orthogonal to the plane through points $A(2,0,-4), B(4,5,8)$ and $C(3,2,7)$. Then find the area of the triangle ABC. (12 marks)

c) Find the vector equation, parametric equation and symmetric equation for line through point $(1,-2,4)$ and is parallel to vector $\langle 4,-6,8 \rangle$. (7 marks)

[TOTAL 30 MARKS]

End of Page.

APPENDIX – KEY FORMULA

Derivatives: $f'(a)=\lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$

Differentiation Rules

General Formulae

1. $\dfrac{d}{dx}[f(x)g(x)]=f(x)g'(x)+g(x)f'(x)$
2. $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right]=\dfrac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$
3. $\dfrac{d}{dx}\left(x^n\right)=nx^{n-1}$
4. $\dfrac{d}{dx}[f(u)]=\dfrac{dy}{du}.\dfrac{du}{dx}$

Exponential and Logarithmic Functions

1. $\dfrac{d}{dx}\left(e^x\right)=e^x$
2. $\dfrac{d}{dx}\left(a^x\right)=a^x\ln a$
3. $\dfrac{d}{dx}(\ln x)=\dfrac{1}{x}$
4. $\dfrac{d}{dx}(\log_a x)=\dfrac{1}{x\ln a}$

Trigonometric Functions

1. $\dfrac{d}{dx}(\sin x)=\cos x$
2. $\dfrac{d}{dx}(\cos x)=-\sin x$
3. $\dfrac{d}{dx}(\tan x)=\sec^2 x$
4. $\dfrac{d}{dx}(\csc x)=-\csc x\cot x$
5. $\dfrac{d}{dx}(\sec x)=\sec x\tan x$
6. $\dfrac{d}{dx}(\cot x)=-\csc^2 x$

Table of Integrals

1. $\int u\,dv=uv-\int v\,du$
2. $\int u^n\,du=\dfrac{u^{n+1}}{n+1}+C,\quad n\neq -1$
3. $\int\dfrac{du}{u}=\ln|u|+C$
4. $\int e^u du=e^u+C$
5. $\int\sin u\,du=-\cos u+C$
6. $\int\cos u\,du=\sin u+C$
7. $\int\sec^2 u\,du=\tan u+C$
8. $\int\csc^2 u\,du=-\cot u+C$
9. $\int\sec u\tan u\,du=\sec u+C$
10. $\int\csc u\cot u\,du=-\csc u+C$

Application of Integrals:

Areas between Curve, $A = \int_a^b [f(x) - g(x)]\,dx$

Differential Equations

Linear Differential Equations

$\frac{dy}{dx} + p(x)y = q(x) \quad \Rightarrow \quad \mu y = \int \mu q(x)\,dx$, where $\mu = e^{\int p(x)\,dx}$

Constant Coefficient of Homogeneous Equations

Roots of Auxiliary Equation, $r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

General Solutions to the Auxiliary Equation:

2 Real & Unequal Roots ($b^2 - 4ac > 0$)	$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$
Repeated Roots ($b^2 - 4ac = 0$)	$y = c_1 e^{rx} + c_2 x e^{rx}$
2 Complex Roots ($b^2 - 4ac < 0$)	$y = e^{ax}(c_1 \cos bx + c_2 \sin bx)$

Constant Coefficient of Non-Homogeneous Equations

$y = y_c + y_p$ [y_c: complementary solution, y_p: particular solution]

Vector

Length of Vector

The length of the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Dot Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}||\mathbf{b}| \cos\theta$

Cross Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$

Area for parallelogram *PQRS*

$= \left| \vec{PQ} \times \vec{PR} \right|$

Area for triangle PQR

$= \frac{1}{2} \left| \vec{PQ} \times \vec{PR} \right|$

Equation of Lines

Vector equation: $\mathbf{r} = \mathbf{r}_0 + \mathbf{tv}$

Parametric equations: $x = x_0 + at$ $y = y_0 + bt$ $z = z_0 + ct$

Symmetric equation: $\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$

Equation of Planes

Vector equation: $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$

Scalar equations: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Linear equation: $ax + by + cz + d = 0$

Angle between Two Planes: $\theta = \cos^{-1}\left(\dfrac{\mathbf{n}_1 \bullet \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|}\right)$